

An Exploration of the Laplace Transform and its Applications

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Abstract

The research question which is under investigation is: How is the Laplace transform more effective when it comes to solving differential equations? A range of sources were consulted and examples are shown to introduce the reader to the Laplace transform and its applications in solving Ordinary Differential Equations (ODEs).

Introduction

In the calculus option for HL mathematics, first order differential equations are taught. This piqued my interest in differential equations as a topic, so after researching more, I found out about the Laplace transform, which is able to solve linear ODEs relatively easily.

To preface this essay, the mathematics within assumes the reader has an understanding of integral calculus and differential equations.

In this essay, differential equations will be introduced briefly first. After that, the Laplace transform will be formally introduced to the reader, followed by convolution. Finally, the applications of the Laplace transform will be shown. In all the above, examples will be used to illustrate how the concepts work to the reader.

Differential Equations

A differential equation is an equation that involves a derivative of a function. According to what was taught in the IB Calculus option, differential equations are described using “ n^{th} order”, and “linear/non-linear”. The term “ n^{th} order” describes the highest order derivative in a differential equation, while linear/non-linear describes the power to which a derivative is raised. An equation is linear if the power is 1, and is non-linear otherwise.